This short document, associated with the book *Curves and Surfaces for Computer Graphics*, shows that even though the tangent vector $\mathbf{P}^t(t)$ of a Hermite curve is smooth inside each segment and is continuous across segments, it is generally non-differentiable at the interior points between the segments.

We start with three given points $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_3$, and three given tangent vectors $\mathbf{P}^t_1$, $\mathbf{P}^t_2$, and $\mathbf{P}^t_3$, at those points. The points can be connected with two Hermite segments $\mathbf{P}_1(t)$ and $\mathbf{P}_2(t)$, whose tangent vectors are given by Equation (4.8), page 116 in the book.

$$\mathbf{P}^t(t) = (t^3, t^2, t, 1)\begin{bmatrix} 0 & 0 & 0 & 0\\ 6 & -6 & 3 & 3\\ -6 & 6 & -4 & -2\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\mathbf{P}_1\\ \mathbf{P}_2\\ \mathbf{P}^t_1\\ \mathbf{P}^t_2\end{bmatrix}. \tag{4.8}$$

Equation (4.8) produces the tangent vector of the first segment as

$$(6t^2 - 6t)\mathbf{P}_1 + (-6t^2 + 6t)\mathbf{P}_2 + (3t^2 - 4t + 1)\mathbf{P}^t_1 + (3t^2 - 2t)\mathbf{P}^t_2.$$

At $t = 1$ (the endpoint of the segment), this tangent has the value $\mathbf{P}^t_2$.

The tangent vector of the second segment is, similarly

$$(6t^2 - 6t)\mathbf{P}_2 + (-6t^2 + 6t)\mathbf{P}_3 + (3t^2 - 4t + 1)\mathbf{P}^t_2 + (3t^2 - 2t)\mathbf{P}^t_3.$$

At $t = 0$ (the start point of this segment) the tangent has the same value $\mathbf{P}^t_2$.

Thus, when the two segments meet at point $\mathbf{P}_2$, their tangent vectors have the same values. We now show that, in general, these tangents approach this value from different directions; the combined tangent vector is therefore non-differentiable at the common point between segments. To show this, we start with Equation (4.9), the second derivative of the Hermite segment

$$\mathbf{P}^{tt}(t) = (t^3, t^2, t, 1)\begin{bmatrix} 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 12 & -12 & 6 & 6\\ -6 & 6 & -4 & -2\end{bmatrix}\begin{bmatrix}\mathbf{P}_1\\ \mathbf{P}_2\\ \mathbf{P}^t_1\\ \mathbf{P}^t_2\end{bmatrix}. \tag{4.9}$$

This equation is both the second derivative of the Hermite segment and the first derivative of its tangent vector.

For the first Hermite curve segment, this equation becomes

$$(12t - 6)\mathbf{P}_1 + (-12t + 6)\mathbf{P}_2 + (6t - 4)\mathbf{P}^t_1 + (6t - 2)\mathbf{P}^t_2.$$

At $t = 1$ (the endpoint of the tangent) its derivative has the value

$$6\mathbf{P}_1 - 6\mathbf{P}_2 + 2\mathbf{P}^t_1 + 4\mathbf{P}^t_2. \tag{1}$$

For the second Hermite curve segment, equation (4.9) becomes

$$(12t - 6)\mathbf{P}_2 + (-12t + 6)\mathbf{P}_3 + (6t - 4)\mathbf{P}^t_2 + (6t - 2)\mathbf{P}^t_3.$$

At $t = 0$ (the start point of this tangent) its derivative has the value

$$-6\mathbf{P}_2 + 6\mathbf{P}_3 - 4\mathbf{P}_2^t - 2\mathbf{P}_3^t. \tag{2}$$

In general, equations (1) and (2) are different. It takes special values of the three point and three tangents to have these equations yield the same value. Thus, we see that the tangent vector of a Hermite curve is generally non-differentiable at the interior points between the segments.

This may not be a problem when Hermite interpolation is used to construct a curve, but when it is applied to surfaces—such as the Catmull-Rom surfaces, page 165—this feature causes different light reflections at the boundaries of surface patches, because the tangent vectors are used to compute the normals to the surface at every point. When the first tangent arrives at point $\mathbf{P}_2$ going in a certain direction and the second tangent leaves this point going in a different direction, the surface normals computed by these tangents around point $\mathbf{P}_2$ will be different, resulting in different amounts of light reflected from both sides of the point. The two surface patches appear to have a sharp (non-smooth) boundary. Figure 1, produced by Peeter Parna, illustrates a typical example of this effect.

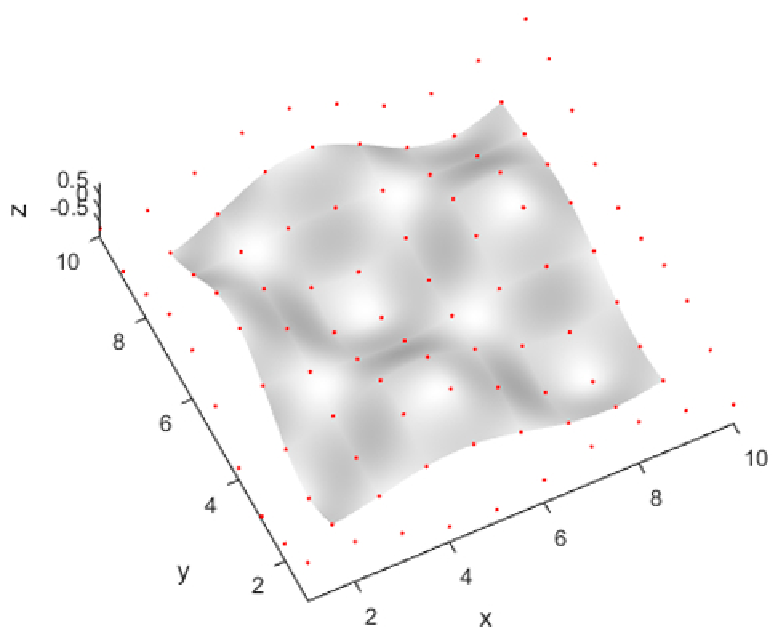

Figure 1: Non-Smooth Boundaries of Catmull-Rom Surface Patches.